Higher order couplings in magnetized brane models

JHEP06(2009)080

(http://iopscience.iop.org/1126-6708/2009/06/080)

Published by IOP Publishing for SISSA

Received: *April 1, 2009*
Accepted: *June 8, 2009*
Published: *June 25, 2009*

Higher order couplings in magnetized brane models

Hiroyuki Abe,[a] Kang-Sin Choi,[b] Tatsuo Kobayashi[b] and Hiroshi Ohki[b]

[a] *Department of Physics, Tohoku University, Sendai 980-8578, Japan*

[b] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

E-mail: abe@tuhep.phys.tohoku.ac.jp, kschoi@gauge.scphys.kyoto-u.ac.jp, kobayash@gauge.scphys.kyoto-u.ac.jp, ohki@scphys.kyoto-u.ac.jp

Abstract: We compute three-point and higher order couplings in magnetized brane models. We show that higher order couplings are written as products of three-point couplings. This behavior is the same as higher order amplitudes by conformal field theory calculations e.g. in intersecting D-brane models.

Keywords: Strings and branes phenomenology, Phenomenology of Field Theories in Higher Dimensions

ArXiv ePrint: 0903.3800

doi:10.1088/1126-6708/2009/06/080

JHEP06(2009)080

Contents

1 Introduction

Extra dimensional field theories, in particular string-derived ones, play important roles in particle physics and cosmology. It is one of keypoints how to realize four-dimensional chiral theories as low-energy effective theories from such higher dimensional theories. Introducing constant magnetic fluxes in extra dimensions is one of interesting scenarios to realize four-dimensional chiral theories [1–10]. Indeed, several models have been studied in field theories and string theories. Furthermore, magnetized D-brane models are T-duals of intersecting D-brane models, and various interesting models have been constructed within the framework of intersecting D-brane models [4–6, 11–13].[1] Orbifolds with magnetic fluxes and other non-trivial backgrounds with magnetic fluxes have also been studied [15–18].

In magnetic background, zero-modes are quasi-localized and the number of zero modes are determined by a size of background magnetic flux. Such a behavior of zero-modes would be important in application for particle phenomenology. Couplings among those zero-modes in four-dimensional effective field theories are obtained as overlap integrals of zero-mode profiles in the extra dimensional space. Thus, if they are localized far away from each other in the extra dimensional space, their four-dimensional couplings would be suppressed and such couplings would be useful to explain suppressed couplings in particle physics such as Yukawa couplings of light quarks and leptons. Hence, computation of those couplings is quite important. Indeed, three-point couplings have been calculated and their results were found to coincide with three-point couplings in intersecting D-brane models [7, 19]. (See also [20].) Furthermore, three-point couplings could lead to realistic Yukawa matrices. (See e.g. [18].)

[1] See for a review [14] and references therein.

For further phenomenological applications, it is also important to compute higher order couplings. Indeed, higher order couplings as well as three-point couplings have been computed within the framework of intersecting D-brane models [21, 22] and heterotic orbifold models [23–27] by using conformal field theory (CFT) technique. Our purpose in this paper is to compute higher order couplings in magnetized brane models. We carry out overlap integrals of three or more wavefunctions in the extra dimensional space in order to obtain higher order couplings in four-dimensional effective field theories. It will be shown that such higher order couplings are written as products of three-point couplings. This behavior is the same as CFT calculations in intersecting D-brane models as well as heterotic orbifold models.

This paper is organized as follows. In section 2, we show our set-up by reviewing ref. [7]. In section 3, we reconsider the computation of the three-point couplings. Its result have been obtained in [7], but here we pay attention to the selection rules and rewrite the result, which is convenient to our purpose. In section 4, we compute the four-point couplings and we study its extensions to higher order couplings. In section 5, we give comments on comparison with those couplings in intersecting D-brane models. Section 6 is devoted to conclusion and discussion.

2 Set-up

We consider dimensional reduction of ten-dimensional $\mathcal{N}=1$ super Yang-Mills theory with $\mathrm{U}(N)$ gauge group [28], on a six torus in Abelian magnetic flux background. We factorize the six-torus into two-tori $(T^2)^3$, each of which is specified by the complex structure τ_d and the area $A_d=(2\pi R_d)^2\ \mathrm{Im}\tau_d$ where $d=1,2,3$. From the periodicity of torus, the background magnetic flux is quantized as [29]

$$F_{z^d\bar{z}^d}=\frac{2\pi i}{\mathrm{Im}\tau_d}\begin{pmatrix} m_1^{(d)}\mathbf{1}_{N_1} & & \\ & \ddots & \\ & & m_n^{(d)}\mathbf{1}_{N_n}\end{pmatrix}, \quad d=1,2,3, \tag{2.1}$$

where $\mathbf{1}_{N_a}$ are the unit matrices of rank N_a, $m_i^{(d)}$ are integers and z^d are the complex coordinates. This background breaks the gauge symmetry $\mathrm{U}(N)\to\prod_{a=1}^{n}\mathrm{U}(N_a)$ where $N=\sum_{a=1}^{n}N_a$.

A magnetic flux in $(4+2n)$ extra dimensions can give rise to chiral fermions in four dimensions. Focusing on a submatrix consisting of two blocks,

$$F_{z^d\bar{z}^d,ab}=\frac{2\pi i}{\mathrm{Im}\tau_d}\begin{pmatrix} m_a^{(d)}\mathbf{1}_{N_a} & 0 \\ 0 & m_b^{(d)}\mathbf{1}_{N_b}\end{pmatrix}, \tag{2.2}$$

the corresponding internal components $\psi_n(z)$ of gaugino fields $\lambda(x,z)$ have the form

$$\lambda(x,z)=\sum_n \chi_n(x)\otimes\psi_n(z), \quad \psi_n(z)=\begin{pmatrix}\psi_n^{aa}(z) & \psi_n^{ab}(z)\\ \psi_n^{ba}(z) & \psi_n^{bb}(z)\end{pmatrix}, \tag{2.3}$$

where x denotes the coordinates of four-dimensional uncompactified space-times, $R^{3,1}$. The off-diagonal components of zero-modes of the Dirac equation transform as bifundamental representations $\psi^{ab} \sim (\mathbf{N_a}, \overline{\mathbf{N}}_\mathbf{b})$, $\psi^{ba} \sim (\overline{\mathbf{N}}_\mathbf{a}, \mathbf{N_b})$ under $\mathrm{SU}(N_a) \times \mathrm{SU}(N_b)$, where we omit the subscript 0 corresponding to the zero-modes, $n = 0$. Since only either of the off-diagonal components has exclusive zero-modes, depending on the sign of the relative magnetic flux $M^{(d)} \equiv m_a^{(d)} - m_b^{(d)}$, the spectrum is chiral; The positive helicity zero-mode provides CPT conjugate to the one with negative helicity. With an appropriate gauge fixing, the zero-modes on each d-th T^2 are written as [7]

$$\psi_d^{j,M^{(d)}}(z^d) = N_{M^{(d)}}\ e^{i\pi M^{(d)} z^d \mathrm{Im}\ z^d/(\mathrm{Im}\ \tau_d)}\ \vartheta \begin{bmatrix} j/M^{(d)} \\ 0 \end{bmatrix} (M^{(d)} z^d, \tau_d M^{(d)}), \tag{2.4}$$

for $j = 1, \ldots, |M^{(d)}|$, where the normalization factor N_M is obtained as

$$N_{M^{(d)}} = \left(\frac{2\mathrm{Im}\tau_d |M^{(d)}|}{A_d^2} \right)^{1/4}. \tag{2.5}$$

We have the $|M^{(d)}|$ zero-modes labelled by the index j. Note that the wavefunction for $j = k + M^{(d)}$ is identical to one for $j = k$. They satisfy the orthonormal condition,

$$\int d^2 z^d\ \psi_d^{i,M^{(d)}}(z^d) \left(\psi_d^{j,M^{(d)}}(z^d) \right)^* = \delta_{ij}. \tag{2.6}$$

The important part of zero-mode wavefunctions is written in terms of the Jacobi theta function

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{n=-\infty}^{\infty} \exp\left[\pi i (n+a)^2 \tau + 2\pi i (n+a)(\nu + b) \right]. \tag{2.7}$$

It transforms under the symmetry of torus lattice and has several important properties [30]. One of them is the following product rule

$$\begin{aligned} &\vartheta \begin{bmatrix} i/M_1 \\ 0 \end{bmatrix} (z_1, \tau M_1) \cdot \vartheta \begin{bmatrix} j/M_2 \\ 0 \end{bmatrix} (z_2, \tau M_2) \\ &\qquad = \sum_{m \in \mathbf{Z}_{M_1+M_2}} \vartheta \begin{bmatrix} \frac{i+j+M_1 m}{M_1+M_2} \\ 0 \end{bmatrix} (z_1 + z_2, \tau(M_1 + M_2)) \\ &\qquad\qquad \times \vartheta \begin{bmatrix} \frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 (M_1+M_2)} \\ 0 \end{bmatrix} (z_1 M_2 - z_2 M_1, \tau M_1 M_2 (M_1 + M_2)). \end{aligned} \tag{2.8}$$

Here $\mathbf{Z}_M$ is the cyclic group of order $|M|$, $\mathbf{Z}_M = \{1, \ldots, |M|\}$ where every number is defined modulo M. Although this expression looks asymmetric under the exchange between i and j, it is symmetric if we take into account the summation. By using the product property (2.8), we can decompose a product of two zero-mode wavefunctions as follows,

$$\begin{aligned} \psi_d^{i,M_1}(z^d) \psi_d^{j,M_2}(z^d) = &\frac{N_{M_1} N_{M_2}}{N_{M_1+M_2}} \sum_{m \in \mathbf{Z}_{M_1+M_2}} \psi_d^{i+j+M_1 m, M_1+M_2}(z^d) \\ &\times \vartheta \begin{bmatrix} \frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 (M_1+M_2)} \\ 0 \end{bmatrix} (0, \tau_d M_1 M_2 (M_1 + M_2)). \end{aligned} \tag{2.9}$$

In this paper, we calculate the generalization of Yukawa couplings to arbitrary order L couplings

$$Y_{i_1 \ldots i_{L_\chi} i_{L_\chi+1} \cdots i_L} \chi^{i_1}(x) \cdots \chi^{i_{L_\chi}}(x) \phi^{i_{L_\chi+1}}(x) \ldots \phi^{i_L}(x), \tag{2.10}$$

with $L = L_\chi + L_\phi$, where χ and ϕ collectively represent four-dimensional components of fermions and bosons, respectively. The system under consideration can be understood as low-energy effective field theory of open string theory. The magnetic flux is provided by stacks of D-branes filling in the internal dimension. The leading order terms in α' are identical to ten-dimensional super-Yang-Mills theory, whose covariantized gaugino kinetic term gives the three-point coupling upon dimensional reduction [7, 19]. The higher order couplings can be read off from the effective Lagrangian of the Dirac-Born-Infeld action with supersymmetrization. The internal component of bosonic and fermionic wavefunctions is the same [7]. Therefore it suffices to calculate the wavefunction overlap in the extra dimensions

$$Y_{i_1 i_2 \ldots i_L} = g_L^{10} \int_{T^6} d^6 z \prod_{d=1}^{3} \psi_d^{i_1,M_1}(z) \psi_d^{i_2,M_2}(z) \ldots \psi_d^{i_L,M_L}(z), \tag{2.11}$$

where g_L^{10} denotes the coupling in ten dimensions.

3 Three-point coupling

In this section, we calculate the three-point coupling considering the coupling selection rule. As we see later, the three-point coupling provides a building block of higher order couplings.

The gauge group dependent part is contracted by the gauge invariance, so that the choice of three blocks m_a, m_b, m_c in (2.1) automatically fixes the relative magnetic fluxes

$$(m_a - m_b) + (m_b - m_c) = (m_a - m_c), \quad \text{and} \quad M_1 + M_2 = M_3, \tag{3.1}$$

where $M_1 = m_a - m_b$, $M_2 = m_b - m_c$ and $M_3 = m_a - m_c$. Here every M_i is assumed to be a positive integer. This relation is interpreted as the selection rule, in analogy of intersecting brane case [31, 32], to which we come back later. If it is not satisfied, there is no corresponding gauge invariant operator in ten dimensions. In terms of quantum numbers the coupling has the form $(\mathbf{N_a}, \overline{\mathbf{N}}_\mathbf{b}, \mathbf{1}) \cdot (\mathbf{1}, \mathbf{N_b}, \overline{\mathbf{N}}_\mathbf{c}) \cdot (\overline{\mathbf{N}}_\mathbf{a}, \mathbf{1}, \mathbf{N_c})$ under $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$.

The internal part including the wavefunction integrals on the d-th T^2 gives

$$y_{ij\bar{k}} = \int d^2 z \; \psi^{i,M_1}(z) \psi^{j,M_2}(z) \left(\psi^{k,M_3}(z) \right)^* . \tag{3.2}$$

The complete three-point coupling is the direct product of those in $d = 1, 2, 3$ and g_3^{10}. For the moment we neglect the normalization factors N_M, and consider two-dimensional wavefunctions, omitting the extra dimensional index d. By using the relation (2.9), we can decompose the product of the first two wavefunctions $\psi^{i,M_1}(z)\psi^{j,M_2}(z)$ in terms of $\psi^{k,M_3}(z)$ and we apply the orthogonality relation (2.6). Then, we obtain

$$y_{ij\bar{k}} = \sum_{m \in \mathbf{Z}_{M_3}} \delta_{i+j+M_1 m, k} \, \vartheta \begin{bmatrix} \frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 M_3} \\ 0 \end{bmatrix} (0, \tau M_1 M_2 M_3), \tag{3.3}$$

where the numbers in the Kronecker delta is defined modulo M_3. This expression is symmetric under the exchange $(i, M_1) \leftrightarrow (j, M_2)$.

For $\gcd(M_1, M_2) = 1$, we solve the constraint from the Kronecker delta $\delta_{i+j+M_1 m,k}$,

$$i + j - k = M_3 l - M_1 m, \quad m \in \mathbf{Z}_{M_3}, l \in \mathbf{Z}_{M_1}. \tag{3.4}$$

Using Euclidean algorithm, it is easy to see that, in the relatively prime case $\gcd(M_1, M_2) = 1$, there is always a unique solution for given i, j, k. This situation is the same as one in intersecting D-brane models [31, 32]. The argument of the theta function in eq. (3.3) becomes

$$\frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 (M_1 + M_2)} = \frac{M_2 k - M_3 j + M_2 M_3 l}{(M_3 - M_2) M_2 M_3}. \tag{3.5}$$

Therefore, the three-point coupling is written as

$$y_{ij\bar{k}}(l) = \vartheta \begin{bmatrix} \frac{M_2 k - M_3 j + M_2 M_3 l}{M_2 M_3 (M_3 - M_2)} \\ 0 \end{bmatrix} (0, \tau(M_3 - M_2) M_2 M_3), \tag{3.6}$$

where l is an integer related to i, j, k through (3.4). This is called the 2-3 picture, or the j-k picture, where the dependence on i and M_1 is only implicit.

In the case with a generic value of $\gcd(M_1, M_2) = g$, we can show

$$y_{ij\bar{k}} = \sum_{n=1}^{g} \vartheta \begin{bmatrix} \frac{M_2 k - M_3 j + M_2 M_3 l}{M_1 M_2 M_3} + \frac{n}{g} \\ 0 \end{bmatrix} (0, \tau M_1 M_2 M_3). \tag{3.7}$$

The point is that, for a given particular solution (i, j, k), the number of general solutions satisfying eq. (3.4) is equal to g. We can use a similar argument as above, now considering $\mathbf{Z}_{M_1/g}$ and $\mathbf{Z}_{M_3/g}$ instead of the original region. There is a unique pair (l, m) in $(\mathbf{Z}_{M_1/g}, \mathbf{Z}_{M_3/g})$ satisfying the constraint (3.4), i.e. ,

$$\frac{i + j - k}{g} = \frac{M_3}{g} l - \frac{M_1}{g} m. \tag{3.8}$$

Obviously, when (l, m) is a particular solution, the following pairs,

$$\left(l + \frac{M_1}{g}, m + \frac{M_3}{g} \right) \in (\mathbf{Z}_{M_1}, \mathbf{Z}_{M_3}), \tag{3.9}$$

also satisfy the equation with the same right-hand side (r.h.s.). Since $\mathbf{Z}_{M_1}$ and $\mathbf{Z}_{M_3}$ are respectively unions of g identical copies of $\mathbf{Z}_{M_1/g}, \mathbf{Z}_{M_3/g}$, there are g different solutions. This situation is the same as one in intersecting D-brane models [31, 32]. If we reflect the shift (3.9) in (3.5), we obtain the desired result (3.7).

There can be Wilson lines $\zeta \equiv \zeta_r + \tau \zeta_i$, whose effect is just a translation of each wavefunction [7]

$$\psi^{j,M}(z) \to \psi^{j,M}(z + \zeta), \quad \text{for all } j. \tag{3.10}$$

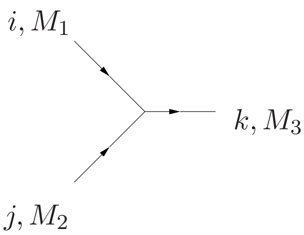

Figure 1. A three-point coupling provides a building block of higher order couplings. This diagram corresponds to the three-point coupling (3.12). The direction of an arrow depends on the holomorphicity of the corresponding external state.

Thus the corresponding product for (2.8) is obtained as

$$\begin{aligned}
&\vartheta \begin{bmatrix} i/M_1 \\ 0 \end{bmatrix} ((z+\zeta_1)M_1, \tau M_1) \cdot \vartheta \begin{bmatrix} j/M_2 \\ 0 \end{bmatrix} ((z+\zeta_2)M_2, \tau M_2) \\
&\qquad = \sum_{m \in \mathbf{Z}_{M_1+M_2}} \vartheta \begin{bmatrix} \frac{i+j+M_1 m}{M_1+M_2} \\ 0 \end{bmatrix} ((M_1+M_2)(z+\zeta_3), \tau(M_1+M_2)) \\
&\qquad\qquad \times \vartheta \begin{bmatrix} \frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 (M_1+M_2)} \\ 0 \end{bmatrix} (M_1 M_2 (\zeta_1 - \zeta_2)), \tau M_1 M_2 (M_1+M_2)),
\end{aligned} \tag{3.11}$$

where $M_3 = M_1 + M_2$ and $\zeta_3 M_3 = \zeta_1 M_1 + \zeta_2 M_2$.

Finally, we take into account the six internal dimensions $T^2 \times T^2 \times T^2$. Referring to (2.11), essentially the full coupling is the direct product of the coupling on each two-torus. The overall factor in (2.11) is the physical ten dimensional gauge coupling $g_3^{10} = g_{\rm YM}$, since this is obtained by dimensional reduction of super Yang-Mills theory. Collecting the normalization factors (2.5) from (2.9), the full three-point coupling becomes

$$\begin{aligned}
Y_{ij\bar{k}} = g_{\rm YM} &\prod_{d=1}^{3} \left(\frac{2{\rm Im}\tau_d}{A_d^2} \frac{M_1^{(d)} M_2^{(d)}}{M_3^{(d)}} \right)^{1/4} \\
&\times \exp\left(i\pi (M_1^{(d)} \zeta_1^{(d)} {\rm Im}\zeta_1^{(d)} + M_2^{(d)} \zeta_2^{(d)} {\rm Im}\zeta_2^{(d)} + M_3^{(d)} \zeta_3^{(d)} {\rm Im}\zeta_3^{(d)}) / {\rm Im}\tau_d \right) \\
&\times \sum_{n_d=1}^{g_d} \vartheta \begin{bmatrix} \frac{M_2^{(d)} k - M_3^{(d)} j + M_2^{(d)} M_3^{(d)} l}{M_1^{(d)} M_2^{(d)} M_3^{(d)}} + \frac{n_d}{g_d} \\ 0 \end{bmatrix} (M_2^{(d)} M_3^{(d)} (\zeta_2^{(d)} - \zeta_3^{(d)}), \tau_d M_1^{(d)} M_2^{(d)} M_3^{(d)}).
\end{aligned} \tag{3.12}$$

Here the index d indicates that the corresponding quantity is the component in d-th direction. For later use, it is useful to visualize the three-point coupling like Feynman diagram in figure 1.

JHEP06(2009)080

4 Higher order coupling

4.1 Four-point coupling

We calculate the four-point coupling

$$y_{ijk\bar{l}} \equiv \int d^2z\ \psi^{i,M_1}(z)\psi^{j,M_2}(z)\psi^{k,M_3}(z)\left(\psi^{l,M_4}(z)\right)^*, \tag{4.1}$$

and represent it in various ways. The main result is that the four-point coupling can be expanded by three-point couplings. Thus by iteration, we can generalize it to higher order couplings.

We consider the case without Wilson lines, since the generalization is straightforward. The product of the first two wavefunctions $\psi^{i,M_1}(z)\psi^{j,M_2}(z)$ in (4.1) is the same as in (2.9). Again, we suppose $M_1+M_2+M_3 = M_4$. Then the product of the first three wavefunctions $\psi^{i,M_1}(z)\psi^{j,M_2}(z)\psi^{k,M_3}(z)$ in (4.1) gives

$$\begin{aligned}\sum_{m\in\mathbf{Z}_{M_1+M_2}}\ \sum_{n\in\mathbf{Z}_{M_4}} \psi^{i+j+k+M_1m+(M_1+M_2)n,M_4}(z)\vartheta\begin{bmatrix}\frac{M_2i-M_1j+M_1M_2m}{M_1M_2(M_1+M_2)}\\ 0\end{bmatrix}(0,\tau M_1M_2(M_1+M_2))\\ \times\vartheta\begin{bmatrix}\frac{M_3(i+j+M_1m)-(M_1+M_2)k+(M_1+M_2)M_3n}{(M_1+M_2)M_3M_4}\\ 0\end{bmatrix}(0,\tau(M_1+M_2)M_3M_4).\end{aligned} \tag{4.2}$$

Now, we product the last wave function $\left(\psi^{l,M_4}(z)\right)^*$ in (4.1), acting on the first factor in (4.2), yielding the Kronecker delta $\delta_{i+j+k+M_1m+(M_1+M_2)n,l}$. The relation is given modulo M_4, reflecting that i,j,k,l are defined modulo M_1,M_2,M_3,M_4, respectively. It is non-vanishing if there is r such that

$$i+j+k+M_1m+(M_1+M_2)n = l+M_4r. \tag{4.3}$$

We solve the constraint equation in terms of n.

For $\gcd(M_1,M_2,M_3)=1$, any coupling specified by (i,j,k,l) satisfies the constraint. For a coupling $y_{ijk\bar{l}}$, fixing (m,r) there is always a unique n satisfying the constraint. This means that by solving the constraint equation in terms of n, we can remove the summation over n in (4.2). The result is

$$y_{ijk\bar{l}} = \sum_{m\in\mathbf{Z}_{M_1+M_2}} \vartheta\begin{bmatrix}\frac{M_2i-M_1j+M_1M_2m}{M_1M_2M}\\ 0\end{bmatrix}(0,\tau M_1M_2M)\cdot\vartheta\begin{bmatrix}\frac{M_3l-M_4k+M_3M_4r}{MM_3M_4}\\ 0\end{bmatrix}(0,\tau M_3M_4M), \tag{4.4}$$

where $M=M_1+M_2=-M_3+M_4$. This form (4.4) is expressed in terms of only 'external lines', i,j,k,l, and in the 'internal line' r is uniquely fixed by m from the relation (4.3). This is to be interpreted as expansion in terms of three-point couplings (3.6). From the property of the theta function, we have relations like $y_{ij\bar{k}} = y^*_{\bar{i}\bar{j}k}$, etc. Thus we can write

$$y_{ijk\bar{l}} = \sum_{m\in\mathbf{Z}_{M_1+M_2}} y_{ij\bar{m}}(m)\cdot y_{km\bar{l}}(r), \tag{4.5}$$

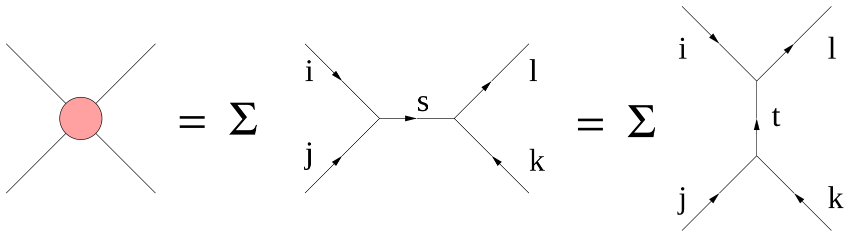

Figure 2. A four-point coupling is decomposed into products of three-point couplings. It also has 'worldsheet' duality. We have another 'u-channel' diagram.

where m and r are uniquely related by the relation (4.3). Recall that three-point coupling can be expressed in terms of 'two external lines' depending on the 2-3 'picture.'

The result (4.4) can be written by arranging the summation of quantum numbers as follows,

$$y_{ijk\bar{l}} = \sum_{s \in \mathbf{Z}_{M_1+M_2}} \vartheta \begin{bmatrix} \frac{M_2 s - Mj + M_2 M r}{M_1 M_2 M} \\ 0 \end{bmatrix} (0, \tau M_1 M_2 M) \cdot \vartheta \begin{bmatrix} \frac{-Ml + M_4 s + M M_4 n}{M_3 M_4 M} \\ 0 \end{bmatrix} (0, \tau M_3 M_4 M). \tag{4.6}$$

Here, we rewrite (4.3)

$$\begin{aligned} i + j + M_1 m &= s + (M_1 + M_2) r, \\ -k + l + M_3 r &= s + (M_1 + M_2) n, \end{aligned} \tag{4.7}$$

by introducing an auxiliary label s, defined modulo $M = M_1 + M_2 = -M_3 + M_4$. This is uniquely fixed by other numbers from (4.3) and it can be traded with m. Thus we arrive at the second form (4.6), which becomes

$$y_{ijk\bar{l}} = \sum_{s \in \mathbf{Z}_{M_1+M_2}} y_{ij\bar{s}} \cdot y_{ks\bar{l}}. \tag{4.8}$$

The second expression (4.6), explicitly depends on the 'internal line' s. It is useful to track the intermediate quantum number s.

We saw that in the case $\gcd(M_1, M_2) = 1$, there is a unique solution. Since we expand higher order coupling in terms of three-point couplings, if any of them have degeneracies as in (3.7), i.e., $\gcd(M_i, M_j) = g_{ij} > 1$, we should take into account their effects. It is interpreted that each three-point coupling contains a flavor symmetry $\mathbf{Z}_{g_{ij}}$ [33]. For the four-point coupling with $\gcd(M_1, M_2) = g_{12}$ and $\gcd(M_3, M_4) = g_{34}$ we have also $\gcd(g_{12}, g_{34}) = g = \gcd(M_1, M_2, M_3, M_4)$, without loss of generality (see below). Employing the 'intermediate state picture', or the (j-s) $\times$ (s-l) picture, in the last expression in (4.6), we have

$$\begin{aligned} \sum_{p \in \mathbf{Z}_g} \sum_{s \in \mathbf{Z}_{M_1+M_2}} \vartheta &\begin{bmatrix} \frac{M_2 s - Mj + M_2 M r}{(M - M_2) M_2 M} + \frac{p}{g} \\ 0 \end{bmatrix} (0, \tau (M - M_2) M_2 M) \\ &\times \vartheta \begin{bmatrix} \frac{-Ml + M_4 s + M M_4 n}{M M_4 (M_4 - M)} + \frac{p}{g} \\ 0 \end{bmatrix} (0, \tau M M_4 (M_4 - M)). \end{aligned} \tag{4.9}$$

It shows that the two symmetries $\mathbf{Z}_{g_{12}}$ and $\mathbf{Z}_{g_{34}}$ are broken down to the largest common symmetry $\mathbf{Z}_g$, due to the constraint. Otherwise we cannot put together the vertices with the common intermediate state s.

Reminding that we are examining the overlap of four wavefunctions, and it *does not depend on the order of product.* If we change the order of the product in (4.1), namely consider the product of the second and the third wavefunctions $\psi^{j,M_2}(z)\psi^{k,M_3}(z)$ first, we have differently-looking constraint relation which is equivalent to (4.3) undergoing the decomposition,

$$\begin{aligned} j+k+M_2 m' &= t+(M_2+M_3)r', \\ -i+l+M_1 r' &= t+(M_2+M_3)n'. \end{aligned} \tag{4.10}$$

This looks like the 't-channel' and we have

$$\begin{aligned} y_{ijk\bar{l}} &= \sum_{t\in\mathbf{Z}_{M'}} \vartheta\begin{bmatrix} \frac{M_3 t - M'k + M_3 M' r'}{(M'-M_3)M_3 M'} \\ 0 \end{bmatrix}(0,\tau(M'-M_3)M_3 M')) \\ &\qquad\qquad \times \vartheta\begin{bmatrix} \frac{-M'l+M_1 t+M' M_1 n}{M' M_1 (M_1-M')} \\ 0 \end{bmatrix}(0,\tau M' M_1 (M_1-M')) \\ &= \sum_{t\in\mathbf{Z}_{M'}} y_{i\bar{l}t}\cdot y_{jk\bar{t}}, \end{aligned} \tag{4.11}$$

with $M' = -M_1 + M_4 = M_2 + M_3$. The result has a behavior like 'worldsheet' *duality* in those of Veneziano and Virasoro-Shapiro [34]. This means that, in decomposing the diagram, the position of an insertion does not matter.

If we have Wilson lines, we just replace the three-point couplings by those with Wilson lines (3.12).

4.2 Generic L-point coupling

We have seen that the four point coupling is expanded in terms of three-point couplings. We can generalize the result to obtain arbitrary higher order couplings. The constraint relations and the higher order couplings are always decomposed into products of three-point couplings. It is easily calculated by Feynman-like diagram.

The decompositions (4.4), (4.6), (4.11) are understood as inserting the identity expanded by the complete set of orthonormal eigenfunctions $\{\psi_n^{i,M}\}$ as follows. For example, we split the integral (4.1) as

$$y_{ijk\bar{l}} = \int d^2z d^2z'\ \psi^{i,M_1}(z)\psi^{j,M_2}(z)\delta^2(z-z')\psi^{k,M_3}(z')\left(\psi^{l,M_4}(z')\right)^*. \tag{4.12}$$

Then, we use the complete set of orthonormal eigenfunctions $\{\psi_n^{i,M}\}$ of the Hamiltonian with a magnetic flux M. That is, they satisfy

$$\sum_{s,n}\left(\psi_n^{s,M}(z)\right)^*\psi_n^{s,M}(z') = \delta^2(z-z'). \tag{4.13}$$

JHEP06(2009)080

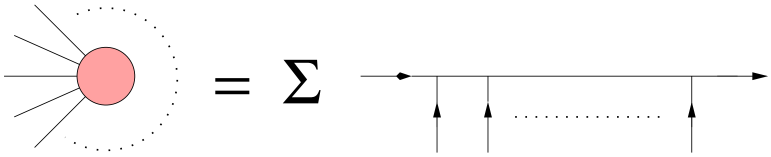

Figure 3. Likewise, any amplitude with arbitrary external lines is decomposed into product of three-point amplitudes.

We insert l.h.s. instead of the delta function $\delta^2(z-z')$ in (4.12). Since $\psi^{i,M_1}(z)\psi^{j,M_2}(z)$ is decomposed in terms of $\psi_n^{s,M_1+M_2}(z)$, it is convenient to take $M=M_1+M_2$ for inserted wavefunctions $\left(\psi_n^{s,M}(z)\right)^*\psi_n^{s,M}(z')$. In such a case, only zero-modes of $\psi_n^{s,M}(z)$ appear in this decomposition. If we take $M\neq M_1+M_2$, higher modes of $\psi_n^{s,M}(z)$ would appear. At any rate, when we take $M=M_1+M_2$, we can lead to the result (4.6) and (4.5). On the other hand, we can split

$$y_{ijk\bar{l}}=\int d^2z d^2z'\ \psi^{j,M_2}(z)\psi^{k,M_3}(z)\delta^2(z-z')\psi^{i,M_1}(z')\left(\psi^{l,M_4}(z')\right)^*, \tag{4.14}$$

and insert (4.13) with $M=M_2+M_3$. Then, we can lead to (4.11). Furthermore, we can calculate the four-point coupling after splitting

$$y_{ijk\bar{l}}=\int d^2z d^2z'\ \psi^{i,M_1}(z)\psi^{k,M_3}(z)\delta^2(z-z')\psi^{j,M_2}(z')\left(\psi^{l,M_4}(z')\right)^*. \tag{4.15}$$

How to split corresponds to 's-channel', 't-channel' and 'u-channel'. Note that only zero-modes appear in 'intermediate states', when we take proper values of M because of the product property.

We have considered the four-point couplings with $M_1+M_2+M_3=M_4$ for $M_i>0$. We may consider the case with $M_1+M_2=M_3+M_4$ for $M_i>0$, which corresponds to

$$y_{ij\bar{k}\bar{l}}\equiv\int d^2z\ \psi^{i,M_1}(z)\psi^{j,M_2}(z)\left(\psi^{k,M_3}(z)\right)^*\left(\psi^{l,M_4}(z)\right)^*. \tag{4.16}$$

In order to consider both of this case and the previous case at the same time, we would have more symmetric expression for the four-point coupling

$$y_{ijkl}=\int d^2z\ \psi^{i_1,M_1}(\tilde{z})\psi^{i_2,M_2}(\tilde{z})\psi^{i_3,M_3}(\tilde{z})\psi^{i_4,M_4}(\tilde{z}), \tag{4.17}$$

by defining

$$\psi^{i,-M}(\bar{z})\equiv\left(\psi^{i,M}(z)\right)^*, \tag{4.18}$$

with

$$M_1+M_2+M_3+M_4=0,$$

where some of M_i are negative, and $\tilde{z}=z$ for $M>0$ and $\tilde{z}=\bar{z}$ for $M<0$.

We can extend the above calculation to the L-point coupling,

$$y_{i_1i_2\ldots i_L}\equiv\int d^2z\prod_{j=1}^{L}\psi^{i_j,M_j}(\tilde{z}), \tag{4.19}$$

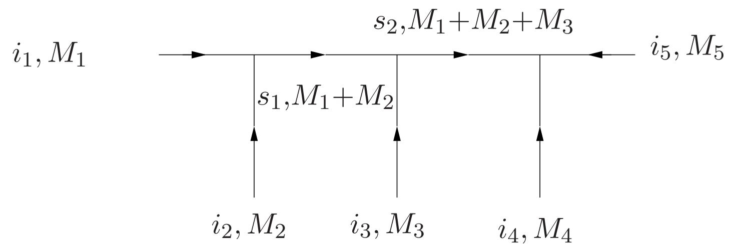

Figure 4. Five-point coupling. No more independent Feynman-like diagram for different insertion.

with the extension as in (4.18). We have then the selection rule

$$\sum_{j=1}^{L} M_j = 0, \tag{4.20}$$

where some of M_j are negative. The constraint is given as

$$\sum_{j=1}^{L} \left(i_j + \left(\sum_{l=1}^{j} M_l \right) r_j \right) = 0. \tag{4.21}$$

Again, it shows the conservation of the total flavor number i_j, reflecting the fact that each i_j is defined modulo M_j. We can decompose L-point coupling into $(L-1)$ and three-point couplings

$$\begin{aligned} \sum_{j=1}^{L-3} \left(i_j + \left(\sum_{l=1}^{j} M_l \right) r_j \right) + i_{L-2} &= s - K r_{L-1}, \\ i_{L-1} + i_L + M_{L-1} r_{L-1} &= -s - K r_{L-2}, \end{aligned} \tag{4.22}$$

where

$$K = \sum_{k=1}^{L-2} M_i = -M_{L-1} - M_L, \tag{4.23}$$

is the intermediate quantum number. Therefore if $\gcd(M_1, M_2, \ldots, M_L) = 1$, by induction we see that there is a unique solution by Euclidean algorithm. By iteration

$$y_{i_1 i_2 \ldots i_L} = \sum_s y_{i_1 i_2 \ldots i_{L-2} s} \cdot y_{\bar{s} i_{L-1} i_L}, \tag{4.24}$$

we can obtain the coupling including the normalization. Thus, we can obtain L-point coupling out of $(L-1)$-point coupling. Due to the independence of ordering, we can insert (or cut and glue) any node.

As an illustrating example we show the result for the five-point coupling. We employ s-channel-like insertions, by naming intermediate quantum numbers s_i as in figure 4. We have

$$
\begin{aligned}
y_{i_1i_2i_3i_4i_5} &= \prod_{j=1}^{5} \vartheta\left[\begin{smallmatrix} i_j/M_j \\ 0 \end{smallmatrix}\right](zM_i, \tau M_i) \\
&= \sum_{s_1,s_2} \vartheta \begin{bmatrix} \frac{M_2 s_1 - (M_1+M_2)i_2 + M_2(M_1+M_2)l_1}{M_2(M_1+M_2)(M_1+2M_2)} \\ 0 \end{bmatrix} (0, M_1M_2(M_1+M_2)\tau) \\
&\quad \times \vartheta \begin{bmatrix} \frac{(M_1+M_2)i_3 - M_3 s_1 + M_3(M_1+M_2)l_2}{M_3(M_1+M_2)(M_1+M_2+M_3)} \\ 0 \end{bmatrix} (0, (M_1+M_2)M_3(M_1+M_2+M_3)) \\
&\quad \times \vartheta \begin{bmatrix} \frac{(M_1+M_2+M_3)i_4 - M_4 s_2 + M_4(M_1+M_2+M_3)l_3}{M_4(M_1+M_2+M_3)(M_1+M_2+M_3+M_4)} \\ 0 \end{bmatrix} (0, -(M_4+M_5)M_4M_5\tau),
\end{aligned}
\tag{4.25}
$$

where

$$
s_1 \in \mathbf{Z}_{M_1+M_2}, \quad s_2 \in \mathbf{Z}_{M_1+M_2+M_3}.
$$

From the regular patterns of increasing orders, we can straightforwardly generalize the couplings to arbitrary order.

Now, taking into account full six internal dimensions, as in three-coupling case (3.12), we have various normalization factors besides the product of theta functions. Again, from the product relation of theta function (2.8) we have

$$
\begin{aligned}
&s_L g_{\rm YM}^{L-2} \alpha'^{(L-4+L_\chi/2)/2} \\
&\times \prod_{d=1}^{3} \left(\frac{2{\rm Im}\tau_d}{A_d^2} \sum_{M_i^{(d)}>0} |M_i^{(d)}| \right)^{-\frac14} \left(\frac{2{\rm Im}\tau_d}{A_d^2} \sum_{M_i^{(d)}<0} |M_i^{(d)}| \right)^{-\frac14} \prod_{i=1}^{L} \left(\frac{2{\rm Im}\tau_d |M_i^{(d)}|}{A_d^2} \right)^{\frac14}.
\end{aligned}
\tag{4.26}
$$

Recall that L_χ is the number of fermions in the couplings (2.10). We have $g_L^{10} = s_L g_{\rm YM}^{L-2} \alpha'^{(L-4+L_\chi/2)/2}$ in (2.11), where symmetric factor s_L comes from higher order expansions of lower-level completion of Yang-Mills theory, having also an expansion parameter α'. In open string theory, it is the Dirac-Born-Infeld action, and it is unknown beyond the quartic order in $\alpha' F$ [36]. The dependence of ten-dimensional gauge coupling $g_{\rm YM}$ and Regge slope α' can be easily accounted by order counting [35]. Note that $g_{\rm YM}$ is dimensionful. This factor (4.26) is non-holomorphic in the complex structure τ and complexified Kähler modulus $\alpha' J = B + iA/4\pi^2$, where $B_{z^d\bar z^d}$ is the antisymmetric tensor field component in d-th two-torus. They are interpreted as originating from the Kähler potential [7, 19]. The product $\prod M_i^{1/4}$ is the leading order approximation of Euler beta function and its multivariable generalization, which is the property of dual amplitude.

As an example of full expressions, we show the four-point coupling among scalar fields, $Y_{ij\bar l\bar m}\phi^i\phi^j(\phi^l)^*(\phi^m)^*$, where ϕ^i and $(\phi^l)^*$ (ϕ^j and $(\phi^m)^*$) correspond to the magnetic flux $M_1^{(d)}$ ($M_2^{(d)}$). For simplicity, we consider the case with vanishing Wilson lines and $\gcd(M_1, M_2) = 1$. The full coupling $Y_{ij\bar{l}\bar{m}}$ is obtained as

$$Y_{ij\bar{l}\bar{m}} = g_{\rm YM}^2 \prod_{d=1}^{3} \left(\frac{2{\rm Im}\tau_d}{A_d^2} \frac{M_1^{(d)} M_2^{(d)}}{M_3^{(d)}} \right)^{1/2} \sum_{k \in \mathbf{Z}_{M_1^{(d)}+M_2^{(d)}}} y^{(d)}_{ij\bar{k}} \, (y^{(d)})^*_{k\bar{l}\bar{m}}, \tag{4.27}$$

up to s_L, where

$$y^{(d)}_{ij\bar{k}} = \vartheta \begin{bmatrix} \frac{M_2^{(d)} k - M^{(d)} j + M_2^{(d)} M^{(d)} r}{M_1^{(d)} M_2^{(d)} M^{(d)}} \\ 0 \end{bmatrix} (0, \tau_d M_1^{(d)} M_2^{(d)} M^{(d)}). \tag{4.28}$$

This scalar coupling with $s_L = 1$ appears from ten-dimensional super Yang-Mills theory and satisfies the relation $Y_{ij\bar{l}\bar{m}} = Y_{ij\bar{k}} (Y)^*_{k\bar{l}\bar{m}}$ for the three-point coupling $Y_{ij\bar{k}}$ in eq. (3.12).

5 Intersecting D-brane models

Here we give comments on the relation between the results in the previous sections and higher order couplings in intersecting D-brane models, i.e. CFT-calculations.

There is well-known T-duality relation between magnetized and intersecting brane models. In intersecting brane case, the wavefunctions are highly localized around intersection points, whereas magnetized brane wavefunctions are fuzzily delocalized over the entire space.

Under the 'horizontal' duality with respect to real axis, $X_z \leftrightarrow 2\pi\alpha' A_z$. The parameter is changed as

$$\tau \leftrightarrow J, \quad \zeta \leftrightarrow \nu. \tag{5.1}$$

Still the translational offset ν is the Wilson line. Thus, the magnetic flux gives the slope $A^i_{\bar{z}} = -\frac{i}{2} F^i_{z\bar{z}} z = \frac{\pi}{{\rm Im}\tau} M_i$ and the corresponding quantum number is the 'relative angle,' for small angles,

$$\pi\theta_i = \frac{M_i}{{\rm Im}J}. \tag{5.2}$$

The selection rule due to the gauge invariance becomes

$$M_1 + M_2 = M_3 \leftrightarrow \theta_1 + \theta_2 = \theta_3. \tag{5.3}$$

In the intersecting brane case, as well as heterotic string case, there have been CFT calculation of higher order amplitude [22, 24, 27] using vertex operator insertion [21, 23, 25, 31]. There are vertex operators V_i corresponding to massless modes. We compute their L-point amplitude,

$$\langle V_1 V_2 \ldots V_L \rangle. \tag{5.4}$$

We have operator product expansion (OPE),

$$V_i(z) V_j(0) \sim \sum_k \frac{c_{ijk}}{z^{h_{ijk}}} V_k(0), \tag{5.5}$$

with $h_{ijk} = h(V_k) - h(V_i) - h(V_j)$, where $h(V_l)$ is the conformal dimension of V_l. This OPE corresponds to (2.9). Furthermore, the coefficients c_{ijk} correspond to the three-point couplings in four-dimensional effective field theory. In ref. [7], it is shown that the above three-point coupling c_{ijk} in intersecting D-brane models corresponds to the T-dual of the three-point couplings Y_{ijk} in magnetized D-brane models.

Now, let us consider the L-point amplitude $\langle \prod_i V_i(z_i) \rangle$. We use the OPE (5.5) to write the L-point amplitude in terms of $(L-1)$ point amplitudes. Such a procedure is similar to one in the previous sections, where we write L-point couplings in terms of three-point couplings.

For example, the CFT calculations for the four-point couplings c_{ijkl} in the intersecting D-brane models would lead

$$c_{ijkl} \sim \sum_s c_{ij\bar{s}} c_{skl}, \tag{5.6}$$

and

$$c_{ijkl} \sim \sum_t c_{ik\bar{t}} c_{tjl}, \tag{5.7}$$

depending on the order of OPE's, i.e. s-channel or t-channel. Thus, the form of the four-point couplings as well as L-point couplings $(L > 4)$ is almost the same as the results in the previous sections. Note that in eq. (2.9), a product of two wavefunctions is decomposed in terms of only the lowest modes. On the other hand, in r.h.s. of eq. (5.5), higher modes as well as lowest modes may appear. However, dominant contribution due to the lowest modes are the same, because c_{ijk} for the lowest modes (i, j, k) corresponds exactly to Y_{ijk} for the lowest modes.

Let us examine the correspondence of couplings between magnetized models and intersecting D-brane models by using concrete formulae. In the intersecting D-brane models, the amplitude (5.4) is decomposed into the classical and the quantum parts,

$$\langle V_1 V_2 \dots V_L \rangle = \mathcal{Z}_{\rm qu} \cdot \mathcal{Z}_{\rm cl} = \mathcal{Z}_{\rm qu} \cdot \sum_{\{X_{\rm cl}\}} \exp(-S_{\rm cl}), \tag{5.8}$$

where $X_{\rm cl}$ is the solution to the classical equation of motion. The classical part is formally characterized as decomposable part and physically gives instanton of worldsheet nature, via the exchange of intermediate string. That gives intuitive understanding via the 'area rule', where the area corresponds to one, which intermediate string sweeps.

In the three-point amplitude, the summation of the classical action $\sum_{\{X_{\rm cl}\}} \exp(-S_{\rm cl})$ becomes the theta function [31], where $S_{\rm cl}$ corresponds to the triangle area. When we exchange τ and J as (5.1) in the magnetized models, the Yukawa coupling (3.6) corresponds to the following expansion

$$\begin{aligned} y_{ij\bar{k}} &= \vartheta \begin{bmatrix} \frac{M_2 k - M_3 j + M_2 M_3 l}{M_1 M_2 M_3} \\ 0 \end{bmatrix} \left(0, iM_1 M_2 M_3 A/(4\pi^2 \alpha')\right) \\ &= \sum_{n \in \mathbf{Z}} \exp\left[-\frac{M_1 M_2 M_3 A}{4\pi\alpha'} \left(\frac{M_2 k - M_3 j + M_2 M_3 l}{M_1 M_2 M_3} + n \right)^2 \right], \end{aligned} \tag{5.9}$$

by using the definition (2.7). We have neglected the antisymmetric tensor component B. The exponent corresponds the area (divided by $4\pi\alpha'$) of possible formation of triangles and

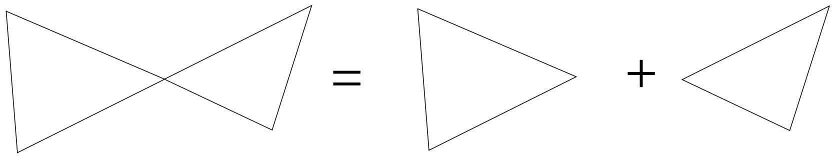

Figure 5. Area of polygon, responsible for the classical part exponent, is decomposed in terms of those of three point functions.

the one with $n = 0$ corresponds to the minimal triangle. Recall that the theta function part depends only τ and J in magnetized and intersecting D-brane models, respectively.

We have omitted the normalization factor, corresponding to the quantum part $\mathcal{Z}_{\rm qu}$. It is obtained by comparing the coupling (5.9) with (3.12). We find the factor

$$2^{-9/4}\pi^{-3}e^{\phi_4/2}\prod_{d=1}^{3}\left(\mathrm{Im}\tau_d\frac{M_1^{(d)}M_2^{(d)}}{M_3^{(d)}}\right)^{1/4}, \tag{5.10}$$

in the magnetized brane side corresponds to

$$\mathcal{Z}_{\rm qu} = (2\pi)^{-9/4}e^{\phi_4/2}\prod_{d=1}^{3}\left((\mathrm{Im}J_d)^2\frac{\theta_1^{(d)}\theta_2^{(d)}}{\theta_3^{(d)}}\right)^{1/4}, \tag{5.11}$$

in the intersecting brane side. We obtain the four dimensional dilaton $\phi_4 = \phi_{10} - \ln|\mathrm{Im}\tau_1\mathrm{Im}\tau_2\mathrm{Im}\tau_3|$ from the ten dimensional one ϕ_{10}, which is related with $g_{\rm YM}$ as $g_{\rm YM} = e^{\phi_{10}/2}\alpha'^{3/2}$. The vacuum expectation value of the dilaton gives gauge coupling $e^{\langle\phi_4\rangle/2} = g$. In this case, the factor containing the angles is a leading order approximation of the ratio of Gamma function

$$\frac{\Gamma(1-\theta_1)\Gamma(1-\theta_2)\Gamma(\theta_3)}{\Gamma(\theta_1)\Gamma(\theta_2)\Gamma(1-\theta_3)} \simeq \frac{\theta_1\theta_2}{\theta_3}, \tag{5.12}$$

valid for small angles. Therefore, the three-point couplings coincide each other between magnetized and intersecting D-brane models. That is the observation of [7].

Now, let us consider the four-point coupling of intersecting D-brane model corresponding to the left figure of figure 5. The four-point amplitude is written as (5.8), where the classical action corresponds to the area of the left figure. However, that can be decomposed into two triangles like the right figure, that is, the classical part can be decomposed into two parts, each of which corresponds to the classical part of three-point amplitude, i.e.

$$\exp(-S_{\rm cl}^{(4)}) = \exp(-S_{\rm cl}^{(3)})\exp(-S'^{(3)}_{\rm cl}), \tag{5.13}$$

where $S_{\rm cl}^{(4)}$ corresponds to the area of the left figure of figure 5 and $S_{\rm cl}^{(3)}$ and $S'^{(3)}_{\rm cl}$ correspond to the triangle areas of the right figure.

On the other hand, our results in the previous sections show that the four-point coupling in the magnetized model is also expanded as (4.6). Each of theta functions in (4.6) corresponds to the classical parts of the three-point couplings in the intersecting D-brane models. This relation corresponds to the above decomposition (5.13). Thus, the theta function parts of the four-point couplings, i.e. the classical part, coincide each other between magnetized and intersecting D-brane models. That means that the holomorphic complex structure, τ, dependence of the four-point couplings in the magnetized brane models is the same as the holomorphic Kähler moduli J dependence in the intersecting D-brane models, since the theta function part in the magnetized (intersecting) D-brane models depends only on τ (J). The other part in the magnetized brane models corresponds to normalization factors N_M. When we take a proper normalization, these factors also coincide.

6 Conclusions

We have calculated three-point and higher order couplings of four-dimensional effective field theory arising from dimensional reduction of magnetized brane models. We have found that higher order couplings are written as products of three-point couplings. This behavior is the same as higher order amplitudes of CFT, that is, higher order amplitudes are decomposed as products of three-point amplitudes in intersecting D-brane models. Our results on higher order couplings would be useful in phenomenological applications. Numerical analysis on higher order couplings is also possible.

Acknowledgments

H. A. is supported by the Grant-in-Aid for the Global COE Program "Weaving Science Web beyond Particle-matter Hierarchy" from the Ministry of Education, Culture, Sports, Science and Technology of Japan. K.-S. C. and T. K. are supported in part by the Grant-in-Aid for Scientific Research No. 20·08326 and No. 20540266 from the Ministry of Education, Culture, Sports, Science and Technology of Japan. T. K. is also supported in part by the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence" from the Ministry of Education, Culture,Sports, Science and Technology of Japan.

References

[1] N.S. Manton, *Fermions and parity violation in dimensional reduction schemes*, *Nucl. Phys.* **B 193** (1981) 502 [SPIRES];
G. Chapline and R. Slansky, *Dimensional reduction and flavor chirality*, *Nucl. Phys.* **B 209** (1982) 461 [SPIRES];
S. Randjbar-Daemi, A. Salam and J.A. Strathdee, *Spontaneous compactification in six-dimensional Einstein-Maxwell theory*, *Nucl. Phys.* **B 214** (1983) 491 [SPIRES];
C. Wetterich, *Dimensional reduction of Weyl, Majorana and Majorana-Weyl spinors*, *Nucl. Phys.* **B 222** (1983) 20 [SPIRES];
P.H. Frampton and K. Yamamoto, *Unitary flavor unification through higher dimensions*, *Phys. Rev. Lett.* **52** (1984) 2016 [SPIRES];
P.H. Frampton and T.W. Kephart, *Left-right asymmetry from the eight sphere*, *Phys. Rev. Lett.* **53** (1984) 867 [SPIRES];

K. Pilch and A.N. Schellekens, *Symmetric vacua of higher dimensional Einstein Yang-Mills theories*, Nucl. Phys. **B 256** (1985) 109 [SPIRES].

[2] E. Witten, *Some properties of $O(32)$ superstrings*, Phys. Lett. **B 149** (1984) 351 [SPIRES];

[3] C. Bachas, *A Way to break supersymmetry*, hep-th/9503030 [SPIRES].

[4] M. Berkooz, M.R. Douglas and R.G. Leigh, *Branes intersecting at angles*, Nucl. Phys. **B 480** (1996) 265 [hep-th/9606139] [SPIRES].

[5] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative compactifications of type-I strings on tori with magnetic background flux*, JHEP **10** (2000) 006 [hep-th/0007024] [SPIRES].

[6] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Type-I strings on magnetised orbifolds and brane transmutation*, Phys. Lett. **B 489** (2000) 223 [hep-th/0007090] [SPIRES].

[7] D. Cremades, L.E. Ibáñez and F. Marchesano, *Computing Yukawa couplings from magnetized extra dimensions*, JHEP **05** (2004) 079 [hep-th/0404229] [SPIRES].

[8] K.-S. Choi, *Unification in intersecting brane models*, Phys. Rev. **D 74** (2006) 066002 [hep-th/0603186] [SPIRES]; *Intersecting brane world from type-I compactification*, Int. J. Mod. Phys. **A 22** (2007) 3169 [hep-th/0610026] [SPIRES].

[9] J. Alfaro, A. Broncano, M.B. Gavela, S. Rigolin and M. Salvatori, *Phenomenology of symmetry breaking from extra dimensions*, JHEP **01** (2007) 005 [hep-ph/0606070] [SPIRES].

[10] J. Troost, *Constant field strengths on $T(2n)$*, Nucl. Phys. **B 568** (2000) 180 [hep-th/9909187] [SPIRES].

[11] G. Aldazabal, S. Franco, L.E. Ibáñez, R. Rabadán and A.M. Uranga, *$D = 4$ chiral string compactifications from intersecting branes*, J. Math. Phys. **42** (2001) 3103 [hep-th/0011073] [SPIRES]; *Intersecting brane worlds*, JHEP **02** (2001) 047 [hep-ph/0011132] [SPIRES].

[12] R. Blumenhagen, B. Körs and D. Lüst, *Type I strings with F- and B-flux*, JHEP **02** (2001) 030 [hep-th/0012156] [SPIRES].

[13] M. Cvetič, G. Shiu and A.M. Uranga, *Three-family supersymmetric standard like models from intersecting brane worlds*, Phys. Rev. Lett. **87** (2001) 201801 [hep-th/0107143] [SPIRES]; *Chiral four-dimensional $N = 1$ supersymmetric type IIA orientifolds from intersecting D6-branes*, Nucl. Phys. **B 615** (2001) 3 [hep-th/0107166] [SPIRES].

[14] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, Ann. Rev. Nucl. Part. Sci. **55** (2005) 71 [hep-th/0502005] [SPIRES]; R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional string compactifications with D-branes, orientifolds and fluxes*, Phys. Rept. **445** (2007) 1 [hep-th/0610327] [SPIRES].

[15] H. Abe, T. Kobayashi and H. Ohki, *Magnetized orbifold models*, JHEP **09** (2008) 043 [arXiv:0806.4748] [SPIRES].

[16] J.P. Conlon, A. Maharana and F. Quevedo, *Wave functions and Yukawa couplings in local string compactifications*, JHEP **09** (2008) 104 [arXiv:0807.0789] [SPIRES].

[17] F. Marchesano, P. McGuirk and G. Shiu, *Open string wavefunctions in warped compactifications*, JHEP **04** (2009) 095 [arXiv:0812.2247] [SPIRES].

[18] H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, *Three generation magnetized orbifold models*, Nucl. Phys. **B 814** (2009) 265 [arXiv:0812.3534] [SPIRES].

[19] P. Di Vecchia, A. Liccardo, R. Marotta and F. Pezzella, *Kähler metrics and Yukawa couplings in magnetized brane models*, *JHEP* **03** (2009) 029 [arXiv:0810.5509] [SPIRES].

[20] R. Russo and S. Sciuto, *The twisted open string partition function and Yukawa couplings*, *JHEP* **04** (2007) 030 [hep-th/0701292] [SPIRES];
D. Duo, R. Russo and S. Sciuto, *New twist field couplings from the partition function for multiply wrapped D-branes*, *JHEP* **12** (2007) 042 [arXiv:0709.1805] [SPIRES].

[21] M. Cvetič and I. Papadimitriou, *Conformal field theory couplings for intersecting D-branes on orientifolds*, *Phys. Rev.* **D 68** (2003) 046001 [*Erratum ibid.* **D 70** (2004) 029903] [hep-th/0303083] [SPIRES];
S.A. Abel and A.W. Owen, *Interactions in intersecting brane models*, *Nucl. Phys.* **B 663** (2003) 197 [hep-th/0303124] [SPIRES];
V. Braun, Y.-H. He and B.A. Ovrut, *Yukawa couplings in heterotic standard models*, *JHEP* **04** (2006) 019 [hep-th/0601204] [SPIRES];
C.-M. Chen, T. Li, V.E. Mayes and D.V. Nanopoulos, *Yukawa corrections from four-point functions in intersecting D6-brane models*, *Phys. Rev.* **D 78** (2008) 105015 [arXiv:0807.4216] [SPIRES].

[22] S.A. Abel and A.W. Owen, *N-point amplitudes in intersecting brane models*, *Nucl. Phys.* **B 682** (2004) 183 [hep-th/0310257] [SPIRES].

[23] S. Hamidi and C. Vafa, *Interactions on orbifolds*, *Nucl. Phys.* **B 279** (1987) 465 [SPIRES];
L.J. Dixon, D. Friedan, E.J. Martinec and S.H. Shenker, *The conformal field theory of orbifolds*, *Nucl. Phys.* **B 282** (1987) 13 [SPIRES].

[24] J.J. Atick, L.J. Dixon, P.A. Griffin and D. Nemeschansky, *Multiloop twist field correlation functions for $Z(N)$ orbifolds*, *Nucl. Phys.* **B 298** (1988) 1 [SPIRES].

[25] T.T. Burwick, R.K. Kaiser and H.F. Muller, *General Yukawa couplings of strings on $Z(N)$ orbifolds*, *Nucl. Phys.* **B 355** (1991) 689 [SPIRES].

[26] S. Stieberger, D. Jungnickel, J. Lauer and M. Spalinski, *Yukawa couplings for bosonic $Z(N)$ orbifolds: their moduli and twisted sector dependence*, *Mod. Phys. Lett.* **A 7** (1992) 3059 [hep-th/9204037] [SPIRES].

[27] K.-S. Choi and T. Kobayashi, *Higher order couplings from heterotic orbifold theory*, *Nucl. Phys.* **B 797** (2008) 295 [arXiv:0711.4894] [SPIRES].

[28] G.F. Chapline and N.S. Manton, *Unification of Yang-Mills theory and supergravity in ten-dimensions*, *Phys. Lett.* **B 120** (1983) 105 [SPIRES];
A.H. Chamseddine, $N = 4$ *supergravity coupled to* $N = 4$ *matter*, *Nucl. Phys.* **B 185** (1981) 403 [SPIRES].

[29] G. 't Hooft, *Some twisted selfdual solutions for the Yang-Mills equations on a hypertorus*, *Commun. Math. Phys.* **81** (1981) 267 [SPIRES];
P. van Baal, SU(N) *Yang-Mills solutions with constant field strength on* T^4, *Commun. Math. Phys.* **94** (1984) 397 [SPIRES];
Z. Guralnik and S. Ramgoolam, *From 0-branes to torons*, *Nucl. Phys.* **B 521** (1998) 129 [hep-th/9708089] [SPIRES].

[30] D. Mumford, *Tata lectures on Theta. Volumes I, II, III*, Progress in Mathematics, volume 28, Birkhäuser Germany (2007).

[31] D. Cremades, L.E. Ibáñez and F. Marchesano, *Yukawa couplings in intersecting D-brane models*, *JHEP* **07** (2003) 038 [hep-th/0302105] [SPIRES].

[32] T. Higaki, N. Kitazawa, T. Kobayashi and K.-J. Takahashi, *Flavor structure and coupling selection rule from intersecting D-branes*, Phys. Rev. **D 72** (2005) 086003 [hep-th/0504019] [SPIRES].

[33] H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, *Non-abelian discrete flavor symmetries from magnetized/intersecting brane models*, arXiv:0904.2631 [SPIRES].

[34] M.A. Virasoro, *Alternative constructions of crossing-symmetric amplitudes withRegge behavior*, Phys. Rev. **177** (1969) 2309 [SPIRES];
J.A. Shapiro, *Narrow-resonance model with Regge behavior for $\pi\pi$ scattering*, Phys. Rev. **179** (1969) 1345 [SPIRES];
G. Veneziano, *Regge intercepts and unitarity in planar dual models*, Nucl. Phys. **B 74** (1974) 365 [SPIRES].

[35] J. Polchinski, *String theory. Volume 1*, Cambridge University Press, Cambridge U.K. (1998).

[36] P. Koerber and A. Sevrin, *The non-abelian D-brane effective action through order α'^4*, JHEP **10** (2002) 046 [hep-th/0208044] [SPIRES].

JHEP06(2009)080